MATH 2 **EXAM #2** **NAME:** Key

Please Note: Use only **one side** of the provided blank papers. Do the problems in order. Use the same guidelines as in your homework assignments. Show **all your work** to backup your answers. Merely stating the answer of a problem results in no credit.

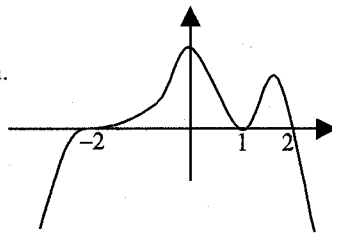
1. Given $f(x)=x^2-1;\ x\le 0$.

a. Find $f^{-1}(x)$.

b. Prove $(f^{-1}\circ f)(x)=x$ (carefully!)

2. A farmer with 10,000 meters of fencing wants to enclose a rectangular field and then divides it into three plots with two fences parallel to one of the sides. What is the dimensions of the largest land that can be enclosed?

3. Find a polynomial with the least degree that can represent the following graph.



4. Sketch the graph of the following polynomial:

$P(x)=-2x^2(x-1)^3(x+1)^2(x-2)$

5. Use the eight legendary steps to sketch the graph of the following rational function. (Number each step clearly as was shown during the lecture)

$$f(x)=\frac{-x^2(x^2-1)}{(x+1)(x^2-4)(x-2)}$$

6. Given $P(x)=x^5+x^4-x^3+x^2+x-2$.

a. Use Des Cartes' rule of sign to predict the number of positive, negative, and complex zeros.

b. Find the smallest upper bound integer and the largest lower bound integer.

7. Find all the real and complex zeros. (Clearly state the candidates, use Des Cartes' rule, upper and lower bound .... Complex zeros must be written in the form of $a+bi$.)

$4x^4+12x^3+19x^2+19x+6=0$

8. Find the polynomial with given zeros, multiplicities and initial condition.

Zero: 1 with the multiplicity of 2

Zero: i+1 with the multiplicity of 1

Zero: i with the multiplicity of 1

$P(0)=2$

9. Simplify each of the following complex expressions and write the answer in the form of $a+bi$. (Show your work. Merely stating the final answer results in no credit.)

a. i^{29}

b. $\dfrac{1}{1+\sqrt{-4}}$

10. (EXTRA CREDIT) Sketch the graph of $y=\left|\log(|x|)\right|$

NOTE: Exponential and logarithmic functions will be covered on the next exam in detail.

① $f(x) = x^2 - 1 \; ; \; x \le 0$

(8) (a) $y = x^2 - 1 \; ; \; x \le 0$

$x = y^2 - 1 \; ; \; y \le 0$

$y^2 = x + 1 \; ; \; y \le 0$

$y = -\sqrt{x+1}$

$\boxed{f^{-1}(x) = -\sqrt{x+1}}$

(7) (b) $(f^{-1} \circ f)(x) = f^{-1}(f(x)) \; ; \; x \le 0$

$= f^{-1}(x^2 - 1) \; ; \; x \le 0$

$= -\sqrt{x^2 + 1}$

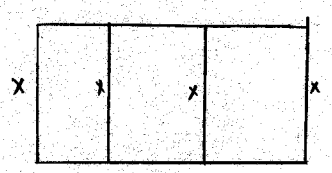
$= -|x| \; ; \; x \le 0$ since $x \le 0 \Rightarrow |x| = -x$

$= -(-x)$

$\boxed{(f^{-1} \circ f)(x) = x}$ Q.E.D.

(2)

(8)



Area → Max

Let x = width.

$4x + 2l = 10{,}000 \Rightarrow \boxed{l = 5{,}000 - 2x}$

Area = A = w·l

$A = x(5000 - 2x)$

$A = -2x^2 + 5000x$

$V\left(\frac{-b}{2a}, ?\right)$

width of largest land = $\frac{-5000}{2(-2)} = \boxed{1{,}250 \text{ m} = w}$

$l = 5000 - 2(1250) \Rightarrow \boxed{l = 2500 \text{ m}}$

(3)

(7)

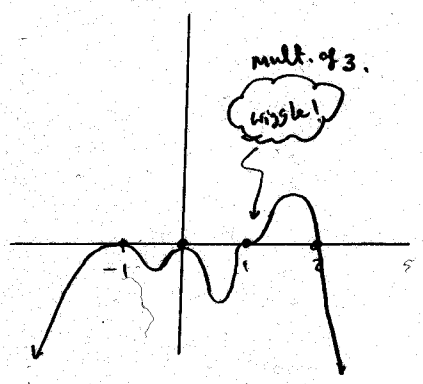
Zeros	min. mult.
−2	3
1	2
2	1

$\Rightarrow \boxed{P(x) = a(x+2)^3(x-1)^2(x-2)}$

(4) $P(x) = -2x^2(x-1)^3(x+1)^2(x-2)$

C.N: 0, 1, −1, 2

x		−1		0		1		2	
P(x)	—	0	—	0	—	0	+	0	−



(5) $f(x) = \dfrac{-x^2(x^2-1)}{(x+1)(x^2-4)(x-2)}$

① $f(x) = \dfrac{-x^2(x-1)(x+1)}{(x+1)(x-2)(x+2)(x-2)}$

② $f(x) = \dfrac{-x^2(x-1)}{(x-2)^2(x+2)}$, $x \neq -1$

③ x-int.: $x = 0, 1$

y-int: $y = 0$

④ VA: $x = 2$ $x = -2$

⑤ HA: $f(x) = \dfrac{-x^3+x^2}{(x^2-4x+4)(x+2)}$

$= \dfrac{-x^3+x^2}{x^3-4x^2+4x+2x^2-8x+8}$

$f(x) = \dfrac{-x^3+x^2}{x^3-2x^2-4x+8}$

since deg. of Num. = deg. of den. $\Rightarrow$ HA: $y = \frac{-1}{1} \Rightarrow y = -1$

⑥ cross HA?

$-1 = \dfrac{-x^3+x^2}{x^3-2x^2-4x+8} \Rightarrow$ $-x^3+x^2 = -x^3+2x^2+4x-8$

$0 = x^2+4x-8 \Rightarrow x = \dfrac{-2 \pm \sqrt{4+8}}{}$

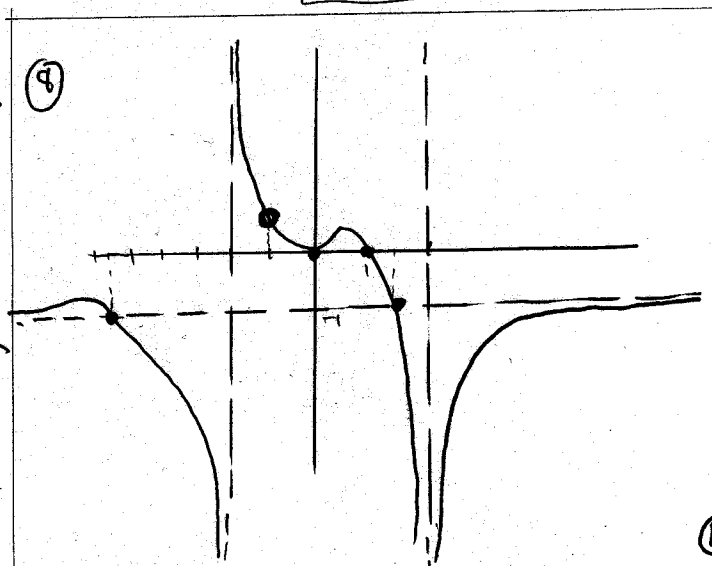
$x = -2+\sqrt{12} \approx 1.5$

$x = -2-\sqrt{12} \approx -5.5$

⑦ CN: 0, 1, 2, −2

x	−∞		−2		0		1		2		+∞
f(x)	−1	—	‖	+	0	+	0	—	‖	—	−1
Graph	below x-axis		Above		Above		below		below		
	HA		VA		x-int		x-int		VA		HA

⑧



⑥ $P(x) = x^5 + x^4 - x^3 + x^2 + x - 2$

ⓐ DesCartes's Rule:

# of pos. zeros: 3 or 1

⑦ $P(-x) = -x^5 + x^4 + x^3 + x^2 - x - 2$

# of neg. zeros: 2 or 0

⇒

Total	5	5	5	5
# of pos. zeros	3	3	1	1
# of neg. zeros	2	0	2	0
# of complx zeros	0	2	2	4

ⓑ Smallest upper bound integer:

⑦

1	1	1	-1	1	1	-2
		1	2	1	2	3
	1	2	1	2	3	1

← all positive ⇒ "1" is an upper bound (smallest upper bound integer).

-1	1	1	-1	1	1	-2
		-1	0	1	-2	1
	1	0	-1	2	-1	-1

not a lower bound (not alternating)

-2	1	1	-1	1	1	-2
		-2	2	-2	2	-6
	1	-1	1	-1	3	-8

← alternating ⇒ "-2" is the largest lower bound integer.

⑦ $4x^4 + 12x^3 + 19x^2 + 19x + 6 = 0$

DesCartes' Rule: # of pos. zeros = None. (Don't use Pos. Cand.)

Rational Candidate: 6: → $\pm1, \pm2, \pm3, \pm6$; 4: → $\pm1, \pm2, \pm4$ ⇒ Cand: ~~-1~~, ~~-2~~, ~~-3~~, ~~-6~~, $-\frac{1}{2}$, $-\frac{3}{2}$, ~~$-\frac{1}{4}$~~, $-\frac{3}{4}$

⑧

Synth. div:

-1	4	12	19	19	6
		-4	-8	-11	-8
	4	8	11	8	$-2 \neq 0$

-2	4	12	19	19	6
		-8	-8	-22	6
	4	4	11	-3	$12 \neq 0$

-3	4	12	19	19	6
		-12	0	-57	114
	4	0	19	-38	120

← alternating ⇒ $x = -3$ is a lower bound. (Don't try -6)

$-\frac{1}{2}$	4	12	19	19	6
		-2	-5	-7	-6
	4	10	14	12	0

⇒ $x = -\frac{1}{2}$

$-\frac{1}{2}$	4	10	14	12
		-2	-4	-5
	4	8	10	$7 \neq 0$

continued on the next page.

(7) continued:

$-\frac{1}{2}$ | 4 | 10 | 14 | 12
| | -1 | $-\frac{9}{4}$ | $-\frac{47}{16}$
| 4 | 9 | $\frac{47}{4}$ | $\neq 0$

$-\frac{3}{2}$ | 4 | 10 | 14 | 12
| | -6 | -6 | -12
| 4 | 4 | 8 | 0

$\Rightarrow \boxed{x = -\frac{3}{2}}$

$4x^2+4x+8=0 \Rightarrow x^2+x+2=0$

$x = \frac{-1 \pm \sqrt{1-4(2)}}{2(1)}$

$x = \frac{-1+\sqrt{-7}}{2}$ $\qquad$ $x = \frac{-1-\sqrt{-7}}{2}$

Zeros: $\boxed{x=-\frac{1}{2}}$, $\boxed{x=-\frac{3}{2}}$, $\boxed{x=-\frac{1}{2}+\frac{\sqrt{7}}{2}i}$, $\boxed{x=-\frac{1}{2}-\frac{\sqrt{7}}{2}i}$

(8) $P(x) = a(x-1)^2(x-(1+i))(x-(1-i))(x-i)(x+i)$

$= a(x-1)^2[(x-1)-i][(x-1)+i](x^2-i^2)$

(8) $= a(x-1)^2[(x-1)^2-i^2](x^2+1)$

$= a(x-1)^2(x^2-2x+1+1)(x^2+1)$

$P(x) = a(x-1)^2(x^2-2x+2)(x^2+1)$

$P(0) = 2 = a(-1)^2(2)(1) \Rightarrow \boxed{a=1}$

$\boxed{P(x) = (x-1)^2(x^2+1)(x^2-2x+2)}$

(9) (a) $i^{29} = i^{28}\,i$

(8) $= (i^2)^{14}\,i$

$= (-1)^{14}\,i$

$= i$

$\boxed{i^{29} = 0+i}$

(b) $\frac{1}{1+\sqrt{-4}} = \frac{1}{1+2i} \cdot \frac{1-2i}{1-2i}$

(8) $= \frac{1-2i}{1-4i^2}$

$= \frac{1-2i}{5}$

$\boxed{\frac{1}{1+\sqrt{-4}} = \frac{1}{5} - \frac{2}{5}i}$

(10 (extra credit) $y = |\log|x||$

+5