1. Solve each of the following equations:

    a. $3^{2x-3} = 2^x$

    b. $\log_2(x) - \log_2(x-2) = 3$

2. Half life of a radioactive material is 2400 years. How long does it take for $\dfrac{1}{3}$ of the radioactive material to decay?

3. Find the exact values of all six trigonometric functions of $30°$ and $60°$ using a right triangle. (You should know these values by heart, but, this question is: "how are those values found?" So, show your proof for these values.)

4. Find two positive and two negative coterminal angles for $\dfrac{9\pi}{4}$.

5. The terminal side of an angle in the second quadrant is parallel to the line $y = -\dfrac{4}{3}x + 2$. Find the exact values of all trigonometric functions of this angle.

6. Find the exact value of each of the followings: (Show your work step by step. Just the final answer is not sufficient. Your work needs to support your answer)

    a. $\cos(120°)$

    b. $\sin\left(\dfrac{7\pi}{6}\right)$

    c. $\tan\left(\dfrac{3\pi}{2}\right)$

    d. $\csc\left(\dfrac{7\pi}{4}\right)$

7. It is given that $\sin(15°) = \dfrac{\sqrt{2-\sqrt{3}}}{2}$. Use this information to find $\sin(345°)$. (Show your work as in number 6)

8. Sketch the graph of each of the following functions. Show at least two cycles (periods).

    a. $y = 3\sin\left(-\pi x + \dfrac{\pi}{3}\right)$

    b. $y = -2\cos\left(2x - \dfrac{\pi}{3}\right)$

9. Solve for θ in general (all solutions) and then hand pick answers in $[-2\pi, \pi]$.

    $\sin(2\theta) = \dfrac{1}{2}$

10. (EXTRA CREDIT) Solve for y: $x = \dfrac{e^y + e^{-y}}{e^y - e^{-y}}$

① ⓐ $3^{2x-3} = 2^x$

ⓑ $\log_3 3^{2x-3} = \log_3 2^x$

$(2x-3)\underbrace{\log_3 3}_{1} = x\underbrace{\log_3 2}_{A}$ (let $A = \log_3 2$)

$2x - 3 = Ax$

$2x - Ax = 3$

$x(2-A) = 3$

$x = \dfrac{3}{2-A}$

$\boxed{x = \dfrac{3}{2 - \log_3 2}}$

ⓑ $\log_2 x - \log_2 (x-2) = 3$

ⓑ $\log_2 \dfrac{x}{x-2} = 3$

$\dfrac{x}{x-2} = 2^3$

$x = 8x - 16$

$7x = 16$

$\boxed{x = \dfrac{16}{7}}$ checked ✓.

② $\begin{cases} A = \frac{1}{2} A_0 \\ t = 2400 \text{ yr.} \end{cases} \xrightarrow{A = A_0 e^{rt}} \quad \frac{1}{2}\cancel{A_0} = \cancel{A_0} e^{r(2400)}$

ⓑ $\ln\left(\frac{1}{2}\right) = 2400\, r$

$\boxed{r = \dfrac{\ln\left(\frac{1}{2}\right)}{2400}}$

$\frac{1}{3}$ decay $\Rightarrow \frac{2}{3}$ remaining.

$\begin{cases} A = \frac{2}{3} A_0 \\ t = ? \end{cases} \Longrightarrow \quad \frac{2}{3}\cancel{A_0} = \cancel{A_0} e^{\frac{\ln\left(\frac{1}{2}\right)}{2400} t}$

$\ln \frac{2}{3} = \dfrac{\ln\left(\frac{1}{2}\right)}{2400} t$

$\boxed{t = \dfrac{2400\, \ln\left(\frac{2}{3}\right)}{\ln\left(\frac{1}{2}\right)}}$

③ start with an equilateral Triangle of sides "2", & draw an altitude

⑧

$h^2 + 1 = 2^2$

$h = \sqrt{3}$

$\sqrt{3} = h$

using the above right Triangle Find all Trig. Funcs.

	$\sin\alpha = \frac{opp}{hyp}$	$\cos\alpha = \frac{adj}{hyp}$	$\tan\alpha = \frac{opp}{Adj}$	$\csc\alpha = \frac{1}{\sin\alpha}$	$\sec\alpha = \frac{1}{\cos\alpha}$	$\cot\alpha = \frac{1}{\tan\alpha}$
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$	2	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$

④ $\frac{9\pi}{4}$:

Positive Coterminal angles: $\frac{9\pi}{4} - 2\pi = \boxed{\frac{\pi}{4}}$

④

$\frac{9\pi}{4} + 2\pi = \boxed{\frac{17\pi}{4}}$

negative Coterminal angles: $\frac{9\pi}{4} - 4\pi = \boxed{\frac{-7\pi}{4}}$

$\frac{9\pi}{4} - 6\pi = \boxed{\frac{-15\pi}{4}}$

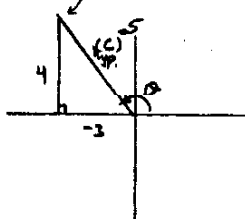⑤ Terminal side parallel to $y = -\frac{4}{3}x + 2 \Rightarrow$ m of Terminal side $= -\frac{4}{3} \leftarrow$ rise / run
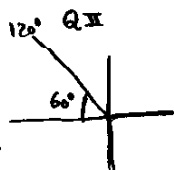
$c^2 = 4^2 + (-3)^2$

$= 16 + 9$

$c^2 = 25$

$\boxed{c = 5}$

⑦

$\boxed{\sin\theta = \frac{4}{5}}$, $\boxed{\cos\theta = \frac{-3}{5}}$, $\boxed{\tan\theta = \frac{4}{3}}$, $\boxed{\cot\theta = \frac{-3}{4}}$, $\boxed{\sec\theta = \frac{-5}{3}}$, $\boxed{\csc\theta = \frac{5}{4}}$
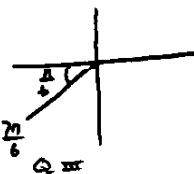
⑥ ⓐ $\cos 120°$

⑥

$\cos 120° = -\cos 60°$

$\boxed{\cos 120° = -\frac{1}{2}}$
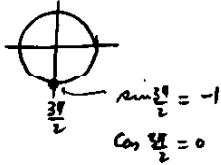
ⓑ $\sin\left(\frac{7\pi}{6}\right)$

⑥ $\sin\frac{7\pi}{6} = -\sin\left(\frac{\pi}{6}\right)$

$\boxed{\sin\frac{7\pi}{6} = -\frac{1}{2}}$

⑥ Continued

© $\tan\left(\frac{3\pi}{2}\right)$

$\sin\frac{3\pi}{2} = -1$

$\cos\frac{3\pi}{2} = 0$

$\tan\left(\frac{3\pi}{2}\right) = \dfrac{\sin\left(\frac{3\pi}{2}\right)}{\cos\left(\frac{3\pi}{2}\right)}$

$= \dfrac{-1}{0}$

$\boxed{\tan\left(\frac{3\pi}{2}\right) \text{ is undefined}}$

⑥ $\csc\left(\frac{7\pi}{4}\right)$

let find $\sin\left(\frac{7\pi}{4}\right)$ first.
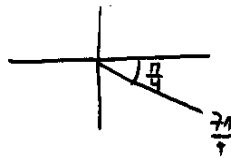

$\frac{\pi}{4}$

$\frac{7\pi}{4}$

$\sin\left(\frac{7\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right)$

$\sin\frac{7\pi}{4} = \dfrac{-\sqrt{2}}{2}$

$\Rightarrow \csc\frac{7\pi}{4} = \dfrac{-2}{\sqrt{2}}\left(\dfrac{\sqrt{2}}{\sqrt{2}}\right)$

$\boxed{\csc\frac{7\pi}{4} = -\sqrt{2}}$

⑦ $\sin(15°) = \dfrac{\sqrt{2-\sqrt{3}}}{2}$

⑧ $\sin(345°) = ?$

$15°$

$345°$

$\sin(345°) = -\sin(15°)$

$\boxed{\sin(345°) = -\dfrac{\sqrt{2-\sqrt{3}}}{2}}$

⑧ ⓐ $y = 3 \sin\left(-\pi x + \frac{\pi}{3}\right)$

$y = 3 \sin\left[-\left(\pi x - \frac{\pi}{3}\right)\right]$

$y = -3 \sin\left(\pi x - \frac{\pi}{3}\right)$

one cycle: $0 \le \pi x - \frac{\pi}{3} \le 2\pi$
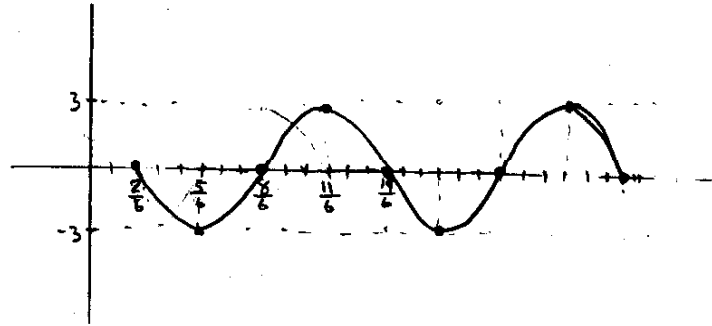
$\frac{\pi}{3} \le \pi x \le \frac{7\pi}{3}$

⑧ $\frac{1}{3} \le x \le \frac{7}{3}$

$\frac{1}{3}$	$\frac{5}{6}$	$\frac{4}{3}$	$\frac{11}{6}$	$\frac{7}{3}$

↓ Common den.

$\frac{2}{6}$	$\frac{5}{6}$	$\frac{8}{6}$	$\frac{11}{6}$	$\frac{14}{6}$
on	MX MN	on	MN MX	on

ⓑ $y = -2 \cos\left(2x - \frac{\pi}{3}\right)$

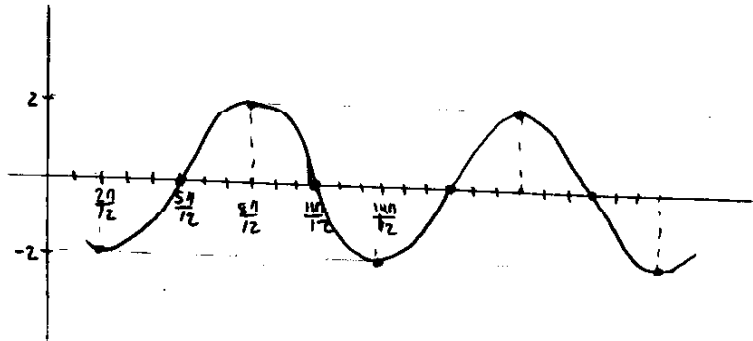one Cycle: $0 \le 2x - \frac{\pi}{3} \le 2\pi$

$\frac{\pi}{3} \le 2x \le \frac{7\pi}{3}$

⑧ $\frac{\pi}{6} \le x \le \frac{7\pi}{6}$

$\frac{\pi}{6}$	$\frac{5\pi}{12}$	$\frac{4\pi}{6}$	$\frac{11\pi}{12}$	$\frac{7\pi}{6}$

$\frac{2\pi}{12}$	$\frac{5\pi}{12}$	$\frac{8\pi}{12}$	$\frac{11\pi}{12}$	$\frac{14\pi}{12}$
MX MN	on	MN MX	on	MX MN

⑨ $\sin(2\theta) = \frac{1}{2}$ → (2θ) is in QI or QII

$\sin(\alpha_R) = \frac{1}{2} \Rightarrow \alpha_R = \frac{\pi}{6}$

$2\theta = \frac{\pi}{6} + n(2\pi)$ $2\theta = \frac{5\pi}{6} + n(2\pi)$

$\boxed{\theta = \frac{\pi}{12} + n\pi}$ $\boxed{\theta = \frac{5\pi}{12} + n\pi}$

$\{\theta \in [-2\pi, \pi]\}$

n	$\frac{\pi}{12} + n\pi$	$\frac{5\pi}{12} + n\pi$
0	$\frac{\pi}{12}$	$\frac{5\pi}{12}$
1		
-1	$\frac{\pi}{12} - \pi = \frac{-11\pi}{12}$	$\frac{5\pi}{12} - \pi = \frac{-7\pi}{12}$
-2	$\frac{\pi}{12} - 2\pi = \frac{-23\pi}{12}$	$\frac{5\pi}{12} - 2\pi = \frac{-19\pi}{12}$

$\theta = \left\{ \frac{\pi}{12}, \frac{5\pi}{12}, \frac{-11\pi}{12}, \frac{-7\pi}{12}, \frac{-23\pi}{12}, \frac{-19\pi}{12} \right\}$