

1. Evaluate each of the following limits. (Show your work. No credit for just the answer.)

a. $\lim_{x \rightarrow -1} \frac{x+1}{x^2-1}$

b. $\lim_{x \rightarrow 2} \frac{\sqrt[5]{x} - \sqrt[5]{2}}{x-2}$

c. $\lim_{x \rightarrow 1} \frac{x^3-1}{x+1}$

d. $\lim_{x \rightarrow 1} \frac{3x-2}{x-1}$

e. $\lim_{x \rightarrow 1} \frac{3x-2}{(x-1)^2}$

f. $\lim_{x \rightarrow 3} \frac{\sqrt[3]{x} - \sqrt[3]{3}}{x-3}$

g. $\lim_{x \rightarrow 2} \frac{x^5-32}{x-2}$

h. $\lim_{x \rightarrow 3} \sqrt{9-x^2}$

i. $\lim_{x \rightarrow -3} \sqrt{9-x^2}$

j. $\lim_{x \rightarrow -2} \sqrt{x-1}$

k. $\lim_{x \rightarrow 4} \sqrt{x-3}$

l. $\lim_{x \rightarrow -1} \frac{|x+1|}{x+1}$

m. $\lim_{x \rightarrow -1} \frac{|x^2-1|}{x^3-1}$

2. Given: $f(x) = \begin{cases} x^3 - x & \text{if } x < -1 \\ \sqrt{1-x^2} & \text{if } |x| < 1, \\ x^2 & \text{if } x \geq 1 \end{cases}$

Find: a. $\lim_{x \rightarrow -1} f(x)$

b. $\lim_{x \rightarrow 1} f(x)$

c. $f(-1)$

d. $f(1)$

e. $f(0)$

f. $f(-2)$

g. $f(3)$

h. $\lim_{x \rightarrow \infty} f(x)$

i. $\lim_{x \rightarrow -\infty} f(x)$

j. $\lim_{x \rightarrow 0} f(x)$

3. Prove each of the following limits:

a. $\lim_{x \rightarrow -1} (3x+2) = -1$

b. $\lim_{x \rightarrow a} (mx+b) = ma+b$

c. $\lim_{x \rightarrow a} c = c$

4. Evaluate each of the following limits.

a. $\lim_{x \rightarrow \infty} \frac{x+5}{\sqrt{x^2-1}}$

b. $\lim_{x \rightarrow -\infty} \frac{x+5}{\sqrt{x^2-1}}$

c. $\lim_{x \rightarrow -\infty} (\sqrt{x^2-1} - x)$

d. $\lim_{x \rightarrow -\infty} \frac{-2x + \sqrt{3x^2-1}}{\sqrt{5x^2+x-3x}}$

e. $\lim_{x \rightarrow \infty} \sin(x)$

f. $\lim_{x \rightarrow \infty} \cos(x)$

g. $\lim_{x \rightarrow -\infty} \frac{(\cos x) - 2x}{3x+1}$

h. $\lim_{x \rightarrow \infty} \left(\frac{1}{x}\right) \sin x$

5. Find all values of x at which each of the following functions are discontinuous and for each value state which requirements of continuity are failed to be satisfied. Label the type of discontinuity.

a. $f(x) = \frac{|x|}{x}$

b. $f(x) = \begin{cases} \sqrt{-x} & \text{if } x < -1 \\ \frac{-1}{x} & \text{if } |x| < 1 \\ x^2+1 & \text{if } x \geq 1 \end{cases}$

6. Show how the definition of limit is developed.

7. Prove that $f(x) = \frac{1}{x^2-4}$ has a vertical asymptote at $x = -2$ and $x = 2$.

8. Prove that $f(x) = \frac{3x^3-2x+5}{2x^2+3x}$ has a horizontal asymptote at $y = \frac{3}{2}$.

9. State the definition of each of the followings:

a. $\lim_{x \rightarrow \infty} f(x) = L$

b. $\lim_{x \rightarrow -\infty} f(x) = L$

c. $\lim_{x \rightarrow -\infty} f(x) = -\infty$

d. $\lim_{x \rightarrow -\infty} f(x) = \infty$