MATH 7 **TAKE HOME QUIZ #2** **DUE: 4-6-2015**

1. Given $f(x)=\begin{cases}2x^2+1 & if \quad x\le 1\\ 4x-2 & if \quad x>1\end{cases}$.

a. **Prove** $f(x)$ is or is not differentiable at $x=1$.

b. Illustrate the results of the side derivative at $x=1$ graphically and describe what kinds of tangent line it has from each side.

2. Use the **definition of derivative** to find the derivative of each of the followings.

a. $f(x)=x^3$ **b.** $f(x)=\sqrt{x+1}$ **c.** $f(x)=\dfrac{1}{x^2}$

3. Use the **definition of derivative** to **prove** that each of the following functions is not differentiable at $x = 1$.

a. $f(x)=|x-1|$ **b.** $f(x)=\begin{cases}x^2+1 & if \quad x>1\\ 2x+3 & if \quad x\le 1\end{cases}$ **c.** $f(x)=\sqrt{x-1}$

4. **Prove** $D_x(c)=0$

5. **Prove** $D_x(x^n)=nx^{n-1}$

6. **Prove** each of the following derivative rule.

a. $D_x[f(x)+g(x)]=f'(x)+g'(x)$ **b.** $D_x[cf(x)]=cf'(x)$

7. Find the equation of the tangent line and the normal line to the graph of the following equation at the given points: $y=x^2+1$

a. at P(1,2) **b*.** at P(0,–1) (notice that this point is not on the graph)

8. **Prove** the Product Rule of Derivative.

9. **Prove** the Quotient Rule of Derivative.

10. Find the derivative of each of the followings using the ***given*** rules and formulas (thus far) only and simplify answer completely.

a. $f(x)=(10-x)^3$ **b.** $y=\left(x^2-x\right)\left(\sqrt{x}\right)$

c. $g(t)=\dfrac{\sqrt[3]{t^2}}{t+1}$

11. Evaluate each of the following limits.

a. $\lim\limits_{x\to 0}\dfrac{\sin^2(3x)}{x\tan(5x)}$ **b.** $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\sin(x)-1}{x-\frac{\pi}{2}}$ **c.** $\lim\limits_{x\to 0}\dfrac{\cos x-1}{x^2}$

12. Prove each of the following derivative formula/rule.

a. $D_x[\cos x]=$ _____ **b.** $D_x[\tan x]=$ _____

c. $D_x[\sec x]=$ _____ **d.** $D_x[\cot x]=$ _____

13. Find the derivative of each of the followings w.r.t. its corresponding independent variable.

a. $y=\dfrac{(\tan x)(\sec x)}{\csc x}$ **b.** $f(x)=\dfrac{\sin x-\cos x}{\sin x}$

c. $g(x)=\sec x\cos x$ **d.** $y=\dfrac{\tan x}{\sec x}$

e. $f(x)=(10-\sin x)^3$ **f.** $y=\sqrt{\left(1-\sqrt{t}\right)^3}$

g. $f(x)=\sin^3\sqrt{2x+1}$ **h.** $f(x)=\sqrt{\sin\left(\sqrt{5+\sqrt{5+\sqrt{5}}}\right)}$

14. Find the second derivative of the following equations.

a. $x^4-y^4=1$ (Simplify completely) **b.** $x^2y^2=y+1$

15. Find the equation of the normal line to the following equation at P(1,1): $x^2+y^2=2$