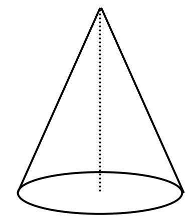
Math 7 **Take Home Quiz #3** **Due: 5-5-15**

1. A right circular cone with a constant radius of 5 cm is inscribed in a sphere. The radius of the sphere is decreasing at the constant rate of 1 cm/min.
a. At what rate is the **height of the cone** changing when the radius of sphere is 14cm.
b. At what rate is the **volume of the cone** increasing when the radius of sphere is 14 cm?

2. A rocket is launched vertically up from a point 10 miles away from an observation station and at the same elevation. If the angle of elevation θ changes at a rate of $2°$ per second when $\theta = 45°$, find the velocity of the rocket at this instant (when $\theta = 45°$).

3*. As sand leaks out of a hole in a container, it forms a conical pile whose altitude is always the same as its radius. If the height of the pile is increasing at a rate of 6 inches per minutes, find the rate at which the sand is leaking out of the container when the altitude is 10 inches.



4. State each of the followings:
a. Definition of an increasing function over an interval I.
b. Definition of a decreasing functions over an interval I.
c. Definition of absolute maximum point and value of *f*.
d. Definition of absolute minimum point and value of *f*.
e. Definition of local (relative) maximum point.
f. Definition of local (relative) minimum point.
g. Definition of a monotonically increasing function.
h. Definition of a monotonically decreasing function.
i. Definition of critical values of a function.
j. Rolle's Theorem
k. Mean Value Theorem (MVT) of derivative.

5. If $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b) and $f'(x) < 0, \forall x \in (a,b)$, then prove that $f(x)$ is decreasing on (a,b).

6. Use the first and second derivative test to sketch the graph of $f(x) = x^{\frac{2}{3}}(x-2)^2$.

7. Use the 8-legendary steps to sketch the graph each of the following functions.
a. $g(x) = \dfrac{\sqrt{x^2-1}}{x}$ **b.** $f(x) = \dfrac{x^2-x-2}{2x^2+3x+1}$ **c.** $f(x) = \dfrac{3x}{x^2+1}$

8. Describe the Newton's Method Algorithm and use this Algorithm to find its corresponding formula.

9. Use the given information to sketch the graph of *f*(x) (use the 8-legendary steps!).

$\lim\limits_{x\to -1^-} f(x) = +\infty$; $\lim\limits_{x\to -1^+} f(x) = -\infty$; $\lim\limits_{x\to 2^-} f(x) = -\infty$; $\lim\limits_{x\to 2^+} f(x) = -\infty$; $\lim\limits_{x\to -\infty} f(x) = 1$;

$\lim\limits_{x\to +\infty} f(x) = 1$; $\lim\limits_{x\to -3} f'(x) = -\infty$; $f'(-2) = f'(0) = f'(1) = 0$; $f''(0) = f''\left(\frac{1}{2}\right) = 0$;

$f'(x) < 0 \quad if \quad x \in (-\infty,-3)\cup(-3,-2)\cup(1,2)$;

$f'(x) > 0 \quad if \quad x \in (-2,-1)\cup(-1,0)\cup(0,1)\cup(2,\infty)$

$f''(x) < 0 \quad if \quad x \in (-\infty,-3)\cup(-1,0)\cup(\tfrac{1}{2},2)\cup(2,\infty)$; $f''(x) > 0 \quad if \quad x \in (-3,-1)\cup(0,\tfrac{1}{2})$

10. $s(t) = t^3 - 3t + 1$ represents the position of a particle moving vertically along a straight line.
a. Find equations representing velocity and acceleration of this particle.
b. Draw a 1-D diagram representing the motion of this particle in the time interval of $[-3,3]$.