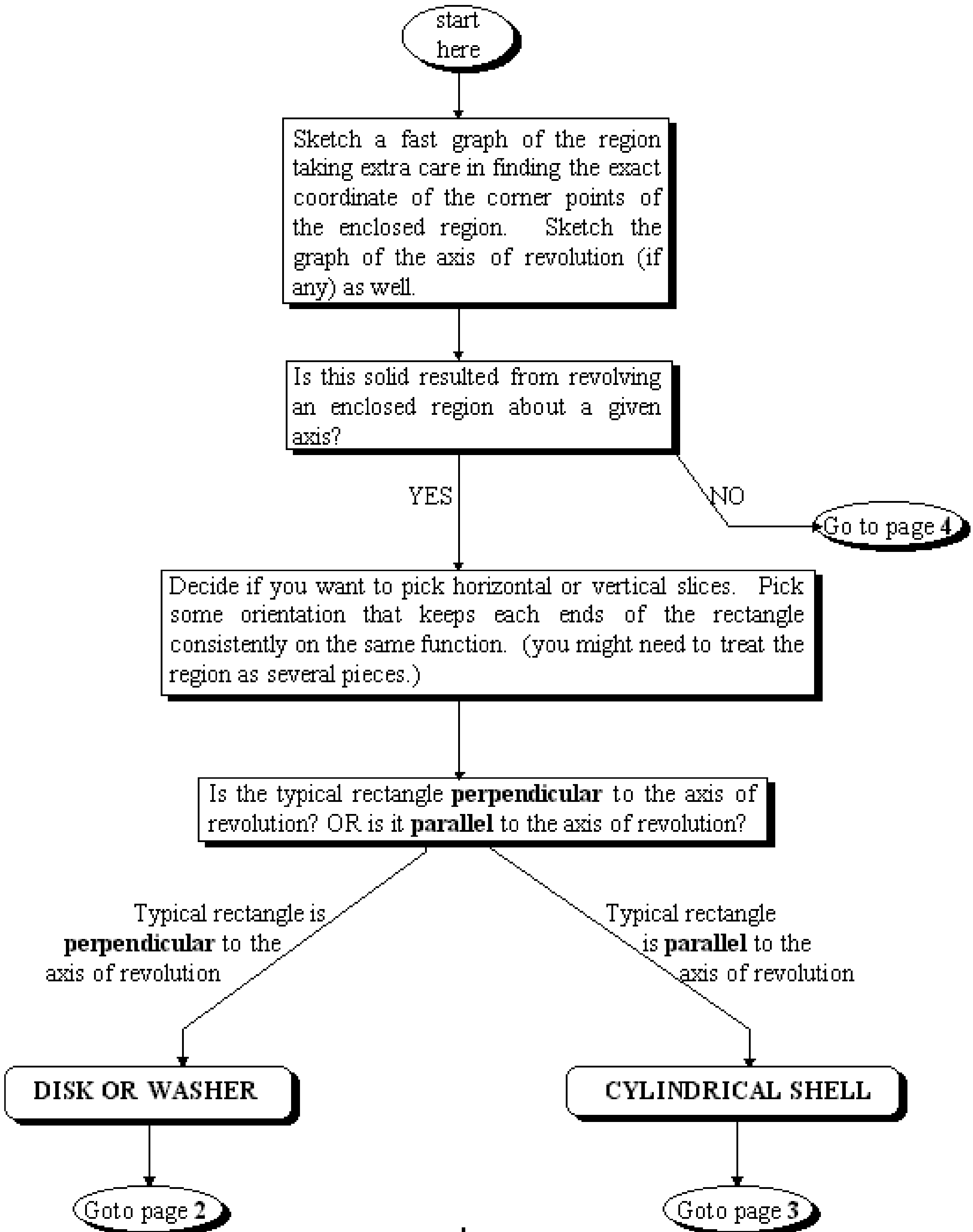
FINDING THE VOLUME OF A SOLID

Volume Master Formula: $\int_a^b \begin{pmatrix}\text{area of the typical} \\ \text{cross-section}\end{pmatrix} \begin{matrix} dx \\ or \\ dy \end{matrix}$



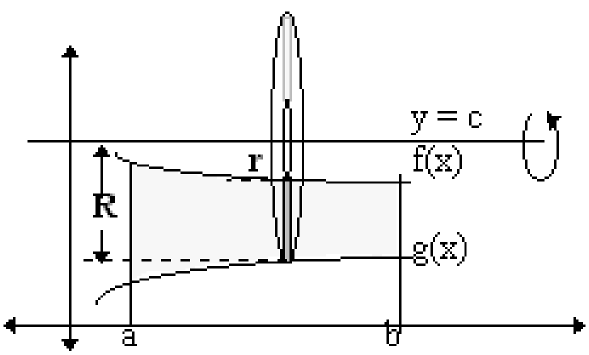
WASHER AND DISK METHOD

Start here

Vertical rectangle

Solve all the equations for y

Draw the revolution of the typical rectangle by drawing its image with respect to the axis of revolution. (axis of revolution acts like a mirror) Connect each points to its image using an orbital curve.



Area of a washer = $\pi(R^2 - r^2)$

R = The radius of the outter circle
r = The radius of the inner circle
(Disk method r = 0)

vertical distance = top − bottom

$$\mathbf{R} = c - g(x)$$
$$\mathbf{r} = c - f(x)$$

area of washer = $\pi\left[(c - g(x))^2 - (c - f(x))^2\right]$

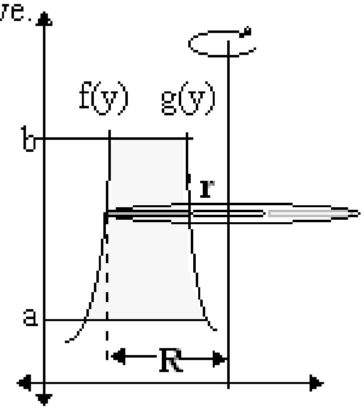
Volume = $\int_a^b \begin{pmatrix}\text{Area of a typical} \\ \text{cross - section}\end{pmatrix} dx$

Volume = $\pi\int_a^b \left[(c - g(x))^2 - (c - f(x))^2\right]dx$

Horizontal rectangle

Solve all the equations for x

Draw the revolution of the typical rectangle by drawing its image with respect to the axis of revolution. (axis of revolution acts like a mirror) Connect each points to its image using an orbital curve.



Area of a washer = $\pi(R^2 - r^2)$

R = The radius of the outter circle
r = The radius of the inner circle
(Disk method r = 0)

hoizantal distance = right − left

$$\mathbf{R} = c - f(y)$$
$$\mathbf{r} = c - g(y)$$

area of washer = $\pi\left[(c - f(y))^2 - (c - g(y))^2\right]$

Volume = $\int_a^b \begin{pmatrix}\text{Area of a typical} \\ \text{cross - section}\end{pmatrix} dy$

Volume = $\pi\int_a^b \left[(c - f(y))^2 - (c - g(y))^2\right]dy$

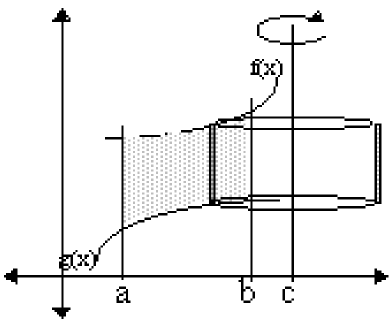
CYLINDRICAL SHELL METHOD

Start here

Vertical rectangle

Solve all the equations for y

Draw the revolution of the typical rectangle by drawing its image with respect to the axis of revolution. (axis of revolution acts like a mirror) Connect each points to its image using an orbital curve. x = c



Area of a Cylindrical Shell = 2π r h

r = The radius of the base (circle)
h = The height of the shell (length of the typical rectangle

Vertical distance = top − bottom
h = f(x) − g(x)

Horizontal distance = right − left
r = c − x

area of a shell = $2\pi(c-x)(f(x)-g(x))$

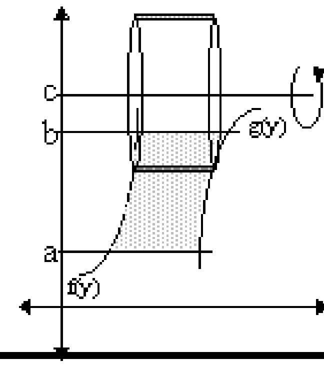
Volume = $\int_a^b \begin{pmatrix}\text{Area of a typical} \\ \text{cross - section}\end{pmatrix} dx$

Volume = $\int_a^b 2\pi(c-x)(f(x)-g(x))dx$

Horizontal rectangle

Solve all the equations for x

Draw the revolution of the typical rectangle by drawing its image with respect to the axis of revolution. (axis of revolution acts like a mirror) Connect each points to its image using an orbital curve.



Area of a Cylindrical Shell = 2π r h

r = The radius of the base (circle)
h = The height of the shell (length of the typical rectangle

Vertical distance = top − bottom
r = c − y

Horizontal distance = right − left
h = g(y) − f(y)

area of a shell = $2\pi(c-y)(g(y)-f(y))$

Volume = $\int_a^b \begin{pmatrix}\text{Area of a typical} \\ \text{cross - section}\end{pmatrix} dy$

Volume = $\int_a^b 2\pi(c-y)(g(y)-f(y))dy$