

FINDING THE VOLUME OF A SOLID

Volume Master Formula: $\int_a^b \left(\begin{array}{l} \text{area of the typical} \\ \text{cross-section} \end{array} \right) \begin{array}{l} dx \\ \text{or} \\ dy \end{array}$

start
here

Sketch a fast graph of the region taking extra care in finding the exact coordinate of the corner points of the enclosed region. Sketch the graph of the axis of revolution (if any) as well.

Is this solid resulted from revolving an enclosed region about a given axis?

YES

NO

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Decide if you want to pick horizontal or vertical slices. Pick some orientation that keeps each ends of the rectangle consistently on the same function. (you might need to treat the region as several pieces.)

Is the typical rectangle **perpendicular** to the axis of revolution? OR is it **parallel** to the axis of revolution?

Typical rectangle is **perpendicular** to the axis of revolution

Typical rectangle is **parallel** to the axis of revolution

DISK OR WASHER

CYLINDRICAL SHELL

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WASHER AND DISK METHOD

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Vertical rectangle

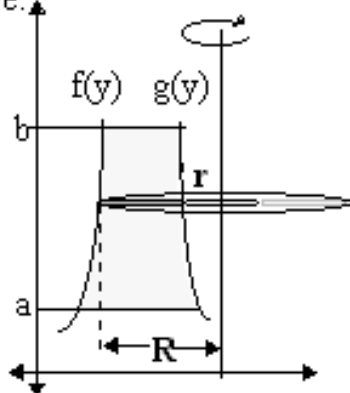
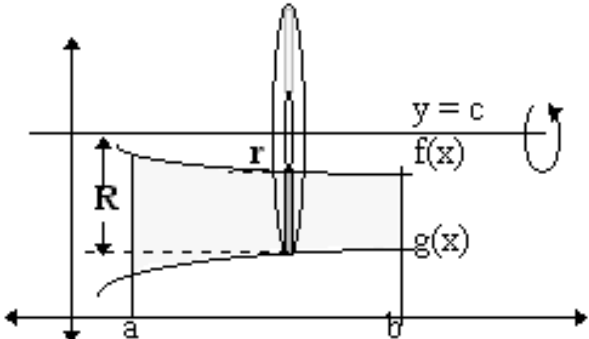
Horizontal rectangle

Solve all the equations for y

Solve all the equations for x

Draw the revolution of the typical rectangle by drawing its image with respect to the axis of revolution. (axis of revolution acts like a mirror) Connect each points to its image using an orbital curve.

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Area of a washer = $\pi(R^2 - r^2)$

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R = The radius of the outer circle
 r = The radius of the inner circle
 (Disk method $r = 0$)

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vertical distance = top - bottom

hoizontal distance = right - left

$$R = c - g(x)$$

$$r = c - f(x)$$

$$R = c - f(y)$$

$$r = c - g(y)$$

area of washer = $\pi[(c - g(x))^2 - (c - f(x))^2]$

area of washer = $\pi[(c - f(y))^2 - (c - g(y))^2]$

Volume = $\int_a^b \left(\text{Area of a typical cross-section} \right) dx$

Volume = $\int_a^b \left(\text{Area of a typical cross-section} \right) dy$

Volume = $\pi \int_a^b [(c - g(x))^2 - (c - f(x))^2] dx$

Volume = $\pi \int_a^b [(c - f(y))^2 - (c - g(y))^2] dy$

CYLINDRICAL SHELL METHOD

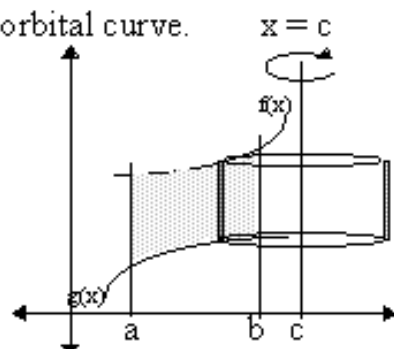
Start here

Vertical rectangle

Horizontal rectangle

Solve all the equations for y

Draw the revolution of the typical rectangle by drawing its image with respect to the axis of revolution. (axis of revolution acts like a mirror) Connect each point to its image using an orbital curve.



Area of a Cylindrical Shell = $2\pi r h$

r = The radius of the base (circle)

h = The height of the shell (length of the typical rectangle)

Vertical distance = top - bottom

$$h = f(x) - g(x)$$

Horizontal distance = right - left

$$r = c - x$$

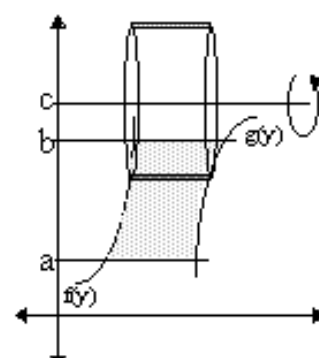
area of a shell = $2\pi(c-x)(f(x)-g(x))$

$$\text{Volume} = \int_a^b \left(\text{Area of a typical cross-section} \right) dx$$

$$\text{Volume} = \int_a^b 2\pi(c-x)(f(x)-g(x))dx$$

Solve all the equations for x

Draw the revolution of the typical rectangle by drawing its image with respect to the axis of revolution. (axis of revolution acts like a mirror) Connect each point to its image using an orbital curve.



Area of a Cylindrical Shell = $2\pi r h$

r = The radius of the base (circle)

h = The height of the shell (length of the typical rectangle)

Vertical distance = top - bottom

$$r = c - y$$

Horizontal distance = right - left

$$h = g(y) - f(y)$$

area of a shell = $2\pi(c-y)(g(y)-f(y))$

$$\text{Volume} = \int_a^b \left(\text{Area of a typical cross-section} \right) dy$$

$$\text{Volume} = \int_a^b 2\pi(c-y)(g(y)-f(y))dy$$