

Evaluate each of the following integrals:

1.  $\int \frac{2x}{1-x^4} dx$

2.  $\int \frac{2x}{1+x^4} dx$

3.  $\int \sin^5 x \cos^3 x dx$

4.  $\int \sin^2 x \cos^4 x dx$

5.  $\int \sec^6 x dx$

6.  $\int \tan^6 x dx$

7.  $\int \tan^3 x dx$

8.  $\int \tan^2 x \sec^3 x dx$

9.  $\int \frac{x}{\sqrt{x^2-9}} dx$

10.  $\int \frac{2}{x^2 \sqrt{1-x^2}} dx$

11.  $\int \frac{1}{(4+x^2)^{\frac{3}{2}}} dx$

12.  $\int \frac{x^3+6x^2+3x+16}{x^3+4x} dx$

13.  $\int \frac{2x^3+10x}{(x^2+1)^2} dx$

14.  $\int \frac{2x+3}{\sqrt{7+6x-x^2}} dx$

15.  $\int \frac{1}{\sqrt[3]{x}-\sqrt{x}} dx$

16. Evaluate each of the following limits.

a.  $\lim_{x \rightarrow 0} \frac{\sin x}{1 - \cos x}$

b.  $\lim_{x \rightarrow \frac{\pi}{2}^+} (\sin x - 1) \tan x$

c.  $\lim_{x \rightarrow 1^+} \left( \frac{1}{x-1} - \frac{1}{x^2-1} \right)$

d.  $\lim_{x \rightarrow 0} \left( 1 + \frac{1}{x^2} \right)^{2x}$

e.  $\lim_{x \rightarrow 1^+} (x-1)^{(x-1)}$

f.  $\lim_{x \rightarrow -\infty} \left[ x + \sqrt{x^2 - 3x + 3} \right]$

17. Prove if the following integral is divergent or convergent:  $\int_1^{+\infty} \frac{1}{x^2+1} dx$

18. Prove that  $\int_1^{\infty} \frac{1}{x^p} dx$  diverges for  $p \leq 1$ .

19. Prove if each of the following sequences converges or diverges. If it converges, determine what it is converging to.

a.  $\left\{ \frac{1}{n^2} \right\}$

b.  $\left\{ \ln \left( \frac{1}{n} \right) \right\}$

c.  $\left\{ \ln \left( \frac{1}{n} + 1 \right) \right\}$

d.  $\{ \sin(n) \}$

e.  $\left\{ \frac{1}{n} \sin(n) \right\}$

f.  $\left\{ \frac{n!}{(n+1)!} \right\}$

g.  $\left\{ \frac{2^{2n}}{3^n} \right\}$

h.  $\left\{ (-1)^n \frac{1}{n^2} \right\}$

i.  $\left\{ (-1)^n \frac{2n}{3n+1} \right\}$

20. Prove if the following series converges or diverges. If it converges, determine what it is converging to (find its sum).

a.  $\sum_{n=1}^{\infty} \frac{n}{2n+1}$

b.  $\sum_{n=1}^{\infty} (-1)^n$

c.  $\sum_{n=1}^{\infty} \sin(n\pi)$

d.  $\sum_{n=1}^{\infty} 3(2)^{-n}$

e.  $\sum_{n=1}^{\infty} \left( \frac{2^n}{3^n} \right)$

f.  $\sum_{n=1}^{\infty} (-1)^n \left( \frac{3^n}{2^{2n}} \right)$

g.  $\sum_{n=1}^{\infty} \ln \left( \frac{1}{n} \right)$

h.  $\sum_{n=1}^{\infty} (-1)^n \frac{2n}{3n+1}$

i.  $\sum_{n=1}^{\infty} \ln \left( \frac{n}{n+1} \right)$

j.  $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$

k.  $\sum_{n=1}^{\infty} \cos \left( \frac{1}{n} \right)$

l.  $\sum (-1)^n \cos(n\pi)$

21. Prove if each of the following series converges or diverges.

a.  $\sum_{n=1}^{\infty} \frac{(2n)!}{n^n}$

b.  $\sum \frac{n^n}{(2n)!}$

c.  $\sum_{n=1}^{\infty} \frac{1}{n^4 + 3n}$

d.  $\sum_{n=1}^{\infty} \frac{n+1}{n^4 + 2}$

e.  $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3 + 2n}}$

f.  $\sum_{n=2}^{\infty} \frac{2^n}{n^n}$

g.  $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

h.  $\sum_{n=1}^{\infty} \frac{n!}{2^n}$

i.  $\sum_{n=1}^{\infty} \frac{3^n + 1}{2^{2n} + 3}$

22. Prove the p-series theorem (that a p-series converges for  $p > 1$  and diverges for  $p \leq 1$ ).

23. Prove the Geometric Series theorem.

24. Prove the Integral Test.

25. Prove the L.C.T. case 1 ( $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L$  where  $L \neq 0$  and  $L$  is finite).