1. Prove whether the following sequences converges or diverges. If it converges, determine what it is converging to.

a. $\left\{\dfrac{2^n}{n!}\right\}$ **b.** $\left\{\dfrac{n!}{2^n}\right\}$

2. Prove if each of the following series is absolutely convergent, conditionally converges or divergent.

a. $\sum_{n=1}^{\infty}(-1)^n\dfrac{1}{n!}$ **b.** $\sum_{n=1}^{\infty}\dfrac{(-2)^n}{n!}$ **c.** $\sum_{n=1}^{\infty}(-1)^n\dfrac{n!}{n^n}$

d. $\sum_{n=2}^{\infty}(-1)^{n-1}\dfrac{n}{\ln n}$ **e.** $\sum_{n=1}^{\infty}(-1)^n\dfrac{3}{\sqrt{n}+1}$

3. Find the radius and the interval of convergence for each of the followings.

a. $\sum_{n=0}^{\infty}n!x^n$ **b.** $\sum_{n=0}^{\infty}\dfrac{(-1)^n}{(2n+1)!}x^{2n+1}$ **c.** $\sum_{n=0}^{\infty}\dfrac{(-1)^n x^n}{\sqrt{n+1}}$

d. $\sum_{n=0}^{\infty}n^2(-3)^n x^n$ **e.** $\sum_{n=0}^{\infty}(-1)^n 2^n(2x-3)^n$ **f.** $\sum_{n=0}^{\infty}\dfrac{x^{2n+1}}{(2n)!}$

4. Given: $\dfrac{1}{1-x}=\sum_{n=0}^{\infty}x^n;\quad x\in(-1,1)$. Use this information to find the power series representation and the radius of convergence for each of the followings.

a. $f(x)=\dfrac{1-x^2}{1+x^2}$ **b.** $f(x)=\dfrac{1}{x}\dfrac{1}{1+x^2}$ **c.** $f(x)=\dfrac{1}{3+x^4}$

d. $f(x)=\dfrac{1}{(1-x)^2}$ **e.** $f(x)=\dfrac{x}{(1+2x)^3}$ **f.** $f(x)=x^2\ln(1+x)$

g. $f(x)=\arctan(2x^2)$ **h.** $f(x)=\ln(2-x)$ **i.** $f(x)=\ln\left(\dfrac{1+x^2}{1-x^2}\right)$

5. Find the Maclaurin series for $f(x)$ using the definition of a Maclaurin series.

a. $f(x)=\sin\left(\dfrac{x}{2}\right)$ **b.** $f(x)=e^{\frac{x}{2}}$ **c.** $f(x)=\dfrac{x}{1+x}$

6. Find the Taylor series for $f(x)$ about the given value of a.

a. $f(x)=\ln(x);\ a=3$ **b.** $f(x)=\sin(2x);\ a=\dfrac{\pi}{8}$ **d.** $f(x)=\sqrt{x};\ a=9$

7. Approximate the following integral to 3 decimal places accuracy: $\int_0^1 \sin x^2 dx$

8. **Setup but don't evaluate** the integral evaluating the surface area obtained by rotating the given curve about the given axis

c. $\{y=x^2-1,\ 0\le x\le 1\}$ about $x=1$ **d.** $\{y=x^2-1,\ 0\le x\le 1\}$ about $y=3$

For problems 9-12:

a. Change each of the following parametric equations to Cartesian equation.
b. Graph the curve represented by each of the parametric equations. (show orientation)
c. Evaluate $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$.

9. $x=\tan t,\quad y=\sec t$ **10.** $x=2\sin t,\quad y=3\cos t$

11. $x=-3\cos^2 t,\ y=2\sin t$ **12.** $x=2\sinh(t),\ y=-3\cosh(t)$

13. Evaluate the surface area generated by revolving $x = \tan t,\ y = \sec t;\ 0 \le t \le \frac{\pi}{4}$ about the line $x = -1$.

14. Prove the surface area formula for a sphere.

For Problems 15-18, **a.** Sketch the graph of each of the following polar equations. **b.** Evaluate $\frac{dy}{dx}$

15. $r = 3\sin(2\theta)$ **16.** $r = 2\cos(3\theta)$ **17.** $r = 5\sec\theta$

18. $r = 2 + 3\sin\theta$